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MACROECONOMIC EFFECTS OF OVER-INVESTMENT IN HOUSING IN AN AGGREGATIVE MODEL OF ECONOMIC ACTIVITY

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Abstract

Is there a theoretical basis for the view that the end of a period of over-investment necessarily leads to a period of below-normal employment as the excess capital stock is run down? We study the repercussions of a false boom in housing driven by prior expectations of future housing prices not justified by fundamentals. When these expectations are corrected, the result is a precipitous drop in housing prices and, on that account alone, some drop in employment. There is also a bulge in the housing stock. In the closed economy case, the downward shift of the term structure of interest rates due to the excess housing stock props up housing prices above the normal steady-state level, so the drop of housing prices "undershoots." Although this transient elevation of housing prices has a positive demand-wage effect on employment, we show that the wealth effect from owning a higher housing stock and a negative Hicks-Lucas-Rapping effect of lower interest rates dominate, so employment drops initially to a below-normal level. The slump gradually subsides as the housing overhang wears off. In the case of a small open economy that faces a world of perfect capital mobility and takes as given the world interest rate, there are two possibilities. If housing services are instantaneously tradeable and perfect substitutes for foreign ones, so purchasing power parity holds, the end of the bubble causes housing prices to drop precisely to the steady-state level. Since there is no undershooting, the wealth effect of the housing overhang is unopposed and the slump is deeper. If domestic and foreign housing services are imperfect substitutes, the country will suffer a period with a weak real exchange rate, thus to a drop of housing prices that "overshoots" the normal level. Here the slump in employment is worsened by the exaggerated fall in housing prices below the steady-state level.
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1. Introduction

Since the modern economy is one that is fraught with novelty, asset prices can turn out to have traveled along disequilibrium paths where expectations of the future turn out to be wrong. This may be one way to describe a "bubble." In this paper, we focus on housing as a major asset. When the bubble comes to an end and the economy finds itself with "too much" housing stock, what does formal theory predict about the level of employment and housing prices in the post-bubble era? In this paper, we aim to answer this basic question. We study this question first in a closed economy and then in a small open economy operating in a world of perfect international capital mobility that takes the world interest rate as given. We find that what happens to aggregate employment and the level of housing prices in the post-bubble era depends on whether we are looking at a closed economy or a small open economy. In the former, the whole term structure of interest rates must adjust to induce individuals in the economy to willingly hold the excess housing stock, which affects housing prices. In the latter, if foreign and domestic housing services are perfectly substitutable, the possibility of leasing the excess stock of housing to foreigners (letting New Yorkers live in London or vice versa) and transfer of home ownership to foreigners allow equilibration to occur at the given world interest rate. However, when domestic and foreign housing services are imperfect substitutes, the equilibration in the face of excess housing stock (assumed to occur only in the home country) requires an adjustment in the real exchange rate with further consequences for the behavior of housing prices and aggregate employment.

More precisely, in the closed economy, we find that the sudden end of a boom due to prior expectations of even higher housing prices in the future that are not ultimately justified by fundamentals results in a precipitous drop in housing prices followed by a period of below-normal aggregate employment—an outright slump. Although housing prices drop precipitously with the end of the bubble, we find that the downward shift of the term structure of interest rates in the closed economy leaves housing prices above the steady-state level so there is undershooting. The unusually low interest rates in the post-bubble era, therefore, serve to prop up housing prices and mitigate the fall in employment via a positive wage effect. Nevertheless, we show that the wealth effect of a higher housing stock on demand for leisure as well as a Hicks-Lucas-Rapping interest-rate effect on labor supply overcome the positive wage effect so the end of the bubble is followed unambiguously by a slump in employment. In the case of a small open economy where foreign and domestic housing services are perfectly substitutable so that purchasing power parity holds, we find that housing prices drop to the steady-state level so there is neither undershooting nor overshooting. Nevertheless, there is still a slump in employment due to a pure wealth effect. However, when domestic and foreign housing services are imperfect substitutes, the possibility of a period of weak real exchange rates caused by the excess housing stock (in the home country) leads to overshooting—a drop of housing prices below the steady-state level—further weakening employment.[1]

The rest of the paper is organized as follows. In section 2, we set up the basic neoclassical model of housing demand and supply where housing stock is the only nonhuman asset and the economy is closed. Then in section 3, we study the small open economy exhibiting perfect

[1]For a careful examination of the data relating to investments and jobs, see Zoega (2010).

international capital mobility. We study two cases in the small open economy, one where foreign and domestic housing services are perfectly substitutable and another where they are imperfect substitutes. Section 4 concludes.

2. The basic closed-economy model of housing, employment, and interest

The economy comprises of individuals who all face a constant and equal probability of death per unit time denoted θ (see Blanchard [1985]).[2] A new cohort is born at each instant of time with individuals facing the same probability of death, θ. The cohort size is sufficiently large that the size of each cohort declines deterministically through time. By normalizing the size of each new cohort to equal θ, total population (also equal to the total size of the labor force) at time t is given by $\int_{-\infty}^{t} \theta \exp^{-\theta(t-s)} ds = 1$. With this normalization, aggregate variables are also interpretable as per-capita variables.

Individuals purchase insurance policies that take the form of receiving payments from the insurance company when alive and turning over their estate to the company upon death. With an actuarially fair scheme, a member who is alive receives the payment θ in exchange for the insurance company's claim of one unit of good contingent on death.

Let the period-t utility function of an individual born at time s be given by $A^{-1} \log(\bar{L} - l(s,t)) + \log h(s,t) + B$. Here, A and B are positive parameters, $\bar{L}$ is total time endowment and $l(s,t)$ is individual labor supply. We note that individuals in this model are not bequeathed with nonhuman wealth at birth and thus start life with a positive supply of labor. As worker-savers, they accumulate nonhuman wealth through their lifetime leaving the possibility that there would exist individuals who become so wealthy that they would use up all their time endowment for leisure. This would make it difficult to derive an aggregate labor supply schedule.[3] To avoid this aggregation problem, we assume that while additional hours spent at market work leads directly to a loss of utility as time available for leisure declines in the traditional way, spending a positive amount of time in market work also gives a positive fixed amount of utility that is subsumed in the term B that the activity of home production does not give. This positive fixed utility from being in the workforce captures the value obtainable

[2] See also Obstfeld (1989) and Weil (1989).
[3] See Hoon (forthcoming) for a discussion of this problem.

from social interactions with colleagues at the workplace as well as mental stimulus from solving problems not normally found from homework. We relegate to the Appendix a formal derivation of the utility function used above as a reduced-form expression from a more basic problem involving utility derivable from consuming a home-produced good.[4]

To ensure that every living person in the economy spends a positive amount of time working in the market in order to facilitate aggregation, we make the following assumption:

Assumption 1: $B > A^{-1}[\log \bar{L} - \log(\bar{L} - 0^+)]$.

Under Assumption 1, a very wealthy individual who might have chosen to retire in a model without a positive utility value from market work spends a very small positive amount of time working in the market ($l_m = 0^+ > 0$) given the positive utility value of market work compared to housework in our model. Setting the factor of proportionality relating the consumption of housing services to the stock of housing equal to one, $h(s,t)$ is the stock of houses giving the equivalent amount of housing services to the individual.

At time 0, the individual's intertemporal optimization problem can be written as

$$\begin{aligned}
&\text{Maximize} \quad \int_0^\infty \{A^{-1}[\log(\bar{L} - l(s,t))] + \log h(s,t) + B\} \exp^{-(\theta+\rho)t}\, dt \\
&\text{subject to} \quad \dot{w}(s,t) = [r(t) + \theta]w(s,t) + v(t)l(s,t) - R(t)h(s,t), \\
&\text{given} \quad w(s,0).
\end{aligned}$$

Here $w(s,t)$ is the individual's nonhuman wealth, $r(t)$ is the interest rate, θ is the actuarially fair premium rate, ρ is the subjective rate of time preference, $v(t)$ is the wage rate, and $R(t)$ is the housing rental rate.

Solving the individual's intertemporal problem gives the following optimal conditions:

$$\frac{A^{-1}}{\bar{L} - l(s,t)} = v(t)\mu(s,t), \tag{1}$$

[4]For a model using an incentive-wage labor market, see Hoon and Phelps (1996).

$$\begin{aligned} \frac{1}{h(s,t)} &= R(t)\mu(s,t), & (2) \\ \frac{\dot{\mu}(s,t)}{\mu(s,t)} &= -[r(t)-\rho], & (3) \\ \lim_{T\to\infty} \mu(s,T)w(s,T)\exp^{-(\theta+\rho)T} &= 0, & (4) \end{aligned}$$

where $\mu(s,t)$ is the co-state variable. Combining (1) and (2), and eliminating $\mu(s,t)$, we obtain

$$h^d(s,t) = \frac{Av(t)[\bar{L} - l^s(s,t)]}{R(t)}. \quad (5)$$

We can interpret (5) as giving individual housing demand ($h^d(s,t)$) as an inverse relation to housing rental ($R(t)$). *Ceteris paribus*, we find that an increase in desired labor supply ($l^s(s,t)$), and a corresponding decline in desired leisure demand, is accompanied by a decline in housing demand. An increase in the wage-rental ratio ($v(t)/R(t)$) also has the effect of boosting housing demand. We choose as our numeraire the standardized unit of rental space and so set the rental rate, $R(t)$, to one in what follows.

Using (5) in the individual's dynamic budget constraint, we obtain

$$\dot{w}(s,t) = [r(t)+\theta]w(s,t) + v(t)\bar{L} - \left(\frac{A+1}{A}\right) h^d(s,t). \quad (6)$$

Using a no-Ponzi game condition that, conditional on being alive at time T, we require

$$\lim_{T\to\infty} \exp^{-\int_t^T [r(\kappa)+\theta]d\kappa} w(s,T) = 0,$$

we can integrate (6) forward in time to obtain

$$\int_0^\infty \left(\frac{A+1}{A}\right) h^d(s,t)\exp^{-\int_0^t [r(\kappa)+\theta]d\kappa} dt = w(s,0) + \int_0^\infty v(t)\bar{L}\exp^{-\int_0^t [r(\kappa)+\theta]d\kappa} dt. \quad (7)$$

The RHS of (7) gives the sum of nonhuman wealth and what may be called full human wealth.

From (2) and (3), we obtain

$$\dot{h}^d(s,t) = [r(t)-\rho]h^d(s,t). \quad (8)$$

Thus individual housing demand rises if the interest rate is above the subjective rate of time preference. Integrating (8) to obtain

$$h^d(s,t) = h^d(s,0)\exp^{\int_0^t [r(\kappa)-\rho]d\kappa},$$

and replacing in (7) gives an individual housing demand function:

$$h^d(s,t) = (\theta+\rho)\left(\frac{A}{A+1}\right)\left[w(s,t) + \int_t^\infty v(\kappa)\bar{L}\exp^{-\int_t^\kappa [r(\nu)+\theta]d\nu} d\kappa\right]. \tag{9}$$

Denoting aggregate housing demand by $H^d(t)$ and aggregate nonhuman wealth by $W(t)$, we obtain the aggregates by integrating over the generations:

$$H^d(t) = \int_{-\infty}^t h^d(s,t)\theta\exp^{-\theta(t-s)} ds, \tag{10}$$

$$W(t) = \int_{-\infty}^t w(s,t)\theta\exp^{-\theta(t-s)} ds. \tag{11}$$

Since the wage rate paid is independent of the cohort, aggregate full human wealth, denoted W^{fh}, is given by

$$W^{fh}(t) = \int_t^\infty v(\kappa)\bar{L}\exp^{-\int_t^\kappa [r(\nu)+\theta]d\nu} d\kappa. \tag{12}$$

From (9) then, we obtain the aggregate housing demand function:

$$H^d(t) = (\theta+\rho)\left(\frac{A}{A+1}\right)[W(t) + W^{fh}(t)]. \tag{13}$$

Differentiating $W(t)$ in (11) with respect to time, we obtain

$$\dot{W}(t) = \theta w(t,t) + \int_{-\infty}^t \left[\theta\exp^{-\theta(t-s)}\dot{w}(s,t) - \theta^2 w(s,t)\exp^{-\theta(t-s)}\right] ds.$$

Taking note that $w(t,t) = 0$ and using (6), this equation can be simplified to give

$$\dot{W}(t) = r(t)W(t) + v(t)\bar{L} - \left(\frac{A+1}{A}\right)H^d(t). \tag{14}$$

Differentiating $W^{fh}(t)$ in (12) with respect to time, we obtain

$$\dot{W}^{fh}(t) = -v(t)\bar{L} + [r(t) + \theta]W^{fh}(t), \tag{15}$$

and

$$\lim_{T\to\infty} W^{fh}(T) \exp^{-\int_t^T [r(\kappa)+\theta]d\kappa} = 0.$$

Using (13), (14) and (15), we obtain

$$\dot{H}^d(t) = [r(t) - \rho]H^d(t) - \theta(\theta + \rho)\left(\frac{A}{A+1}\right)W(t). \tag{16}$$

We also note that aggregating across generations in (5), using the normalization $R(t) = 1$, gives

$$H^d(t) = Av(t)[\bar{L} - L^s(t)]. \tag{17}$$

Turning to the production side of the model, we assume that the construction sector is perfectly competitive. Only labor is required to produce a new house or replace an aging one. With linear technology given by $\Lambda L(t)$, the following optimal condition must be satisfied if there is positive construction activity:

$$q(t)\Lambda = v(t), \tag{18}$$

where $q(t)$ is the price of housing (our asset price) measured in units of standardized rental space. Net investment in housing is given by

$$\dot{H}(t) = \Lambda L(t) - \delta H(t), \tag{19}$$

where δ is the rate of depreciation of housing stock.

In the closed economy, total housing demand must equal to the stock of housing available, that is, $H^d(t) = H(t)$. Using this condition in (16) and taking note that nonhuman wealth

$W(t)$ is equal to $q(t)H(t)$, we obtain an expression for the real interest rate:

$$r(t) = \rho + \theta(\theta + \rho)\left(\frac{A}{A+1}\right)q(t) + \frac{\dot{H}(t)}{H(t)}. \tag{20}$$

For intertemporal equilibrium, the following arbitrage condition must hold:

$$\begin{aligned} r(t) &= \frac{R(t)}{q(t)} + \frac{\dot{q}(t)}{q(t)} - \delta \\ &= \frac{1}{q(t)} + \frac{\dot{q}(t)}{q(t)} - \delta, \end{aligned} \tag{21}$$

where we use the normalization $R(t) = 1$.

Rewriting (19) as

$$\dot{H}(t) = \Lambda\bar{L} - \Lambda[\bar{L} - L(t)] - \delta H(t),$$

and then using (17), (18) and the condition $H^d(t) = H(t)$, we make net investment in housing a function of $q(t)$ and $H(t)$:

$$\dot{H}(t) = \Lambda\bar{L} - \frac{H(t)}{Aq(t)} - \delta H(t). \tag{22}$$

Notice that by substituting out for $\dot{H}(t)$ in (20) using (22), we obtain an expression for $r(t)$ in terms of $H(t)$ and $q(t)$:

$$r(t) = \rho + \theta(\theta + \rho)\left(\frac{A}{A+1}\right)q(t) + \frac{\Lambda\bar{L}}{H(t)} - \frac{1}{\Lambda q(t)} - \delta. \tag{23}$$

From (23), we infer that, *ceteris paribus*, $r(t)$ increases with $q(t)$ but decreases with $H(t)$.

Equating the expressions for $r(t)$ in (20) and (21), and using (22), we obtain the dynamic equation that gives the evolution of $q(t)$ over time:

$$\dot{q}(t) = \left\{\rho + \theta(\theta + \rho)\left(\frac{A}{A+1}\right)q(t)\right\}q(t) + \frac{q(t)\Lambda\bar{L}}{H(t)} - \left(\frac{A+1}{A}\right). \tag{24}$$

Equations (22) and (24) give us the pair of dynamic equations tracing the general-equilibrium behavior of the economy given an initial stock of housing, $H(0)$.

Around the steady state, the slope of the stationary-H locus is given by

$$\left.\frac{dq(t)}{dH(t)}\right|_{\dot{H}(t)=0} = \frac{\frac{\Lambda\bar{L}}{H_{ss}}}{\frac{H_{ss}}{A(q_{ss})^2}}. \tag{25}$$

In turn, the slope of the stationary-q locus is given by

$$\left.\frac{dq(t)}{dH(t)}\right|_{\dot{q}(t)=0} = \frac{\left(\frac{1}{A+1}\right)\left[\frac{\Lambda\bar{L}}{H_{ss}}\right]}{\left[\frac{A}{(A+1)^2}\right]\theta(\theta+\rho)H_{ss} + \frac{H_{ss}}{A(q_{ss})^2}}. \tag{26}$$

Inspecting (25) and (26), we see that

$$\left.\frac{dq(t)}{dH(t)}\right|_{\dot{H}(t)=0} > \left.\frac{dq(t)}{dH(t)}\right|_{\dot{q}(t)=0} > 0.$$

The phase diagram characterizing the general-equilibrium evolution of the economy is shown in Figure 1, where it is readily checked that the system exhibits saddle-path stability. The saddle path is positively sloped and has the smallest gradient.

The result that the saddle path is positively sloped can be better understood by conducting the following hypothetical exercise. Starting from an initial steady state, suppose that individuals waking up in the morning suddenly discover that they have been bequeathed with a larger stock of houses. The economy finds itself with an excess stock of houses. How does the economy respond? From (17), there are two ways to boost aggregate housing demand to equal the suddenly discovered higher stock of houses. The first is to contract labor supply or increase leisure demand to enjoy the higher stock of houses; the second is for the wage-rental ratio or, given our normalization, for the real wage measured in terms of standardized units of rental space to rise in order to make it affordable for individuals to live in the extra space. From (18), we find that the only way for real wages to rise is for housing prices to rise (relative to the initial steady-state level). From (21), we have, integrating forward,

$$q(t) = \int_t^{\infty} 1 \exp^{-\int_t^{\kappa}[r(\kappa)+\delta](\kappa-t)}\, d\kappa. \tag{27}$$

The stream of current and future rentals, equal to one in each period by our normalization, must be discounted by current and future instantaneous real rates of interest. The positive slope of the saddle path means that the sudden discovery of an excess stock of houses (above the initial steady-state level) results in a sudden rise in q with the expectation formed that housing prices will then gradually decline. From (21), we infer that the sudden rise in q accompanied by an expectation of gradually declining asset prices back to the original steady-state level (absent any other unforseen shock) indeed corresponds to a downward shift of the term structure of interest rates.

Using (22) with $\dot{H}(t) = 0$ and (24) with $\dot{q}(t) = 0$, we obtain an equation implicitly determining the steady-state value of q denoted q_{ss}:

$$\rho + \delta + \theta(\theta + \rho)\left[\frac{A}{A+1}\right] q_{ss} = \frac{1}{q_{ss}}. \tag{28}$$

We infer from (28) that q_{ss} is decreasing in the following parameters, the model's "fundamentals" that drive housing prices: ρ, θ, δ and A.

Lemma 1: $q_{ss} = \mathrm{Q}(\rho, \theta, \delta, A)$ with $\mathrm{Q}_1 < 0;\ \mathrm{Q}_2 < 0;\ \mathrm{Q}_3 < 0;\ \mathrm{Q}_4 < 0$.

The steady-state level of housing stock is readily inferred from the relation

$$H_{ss} = \frac{\Lambda \bar{L}}{\left[\frac{1}{A q_{ss}} + \delta\right]}.$$

Since the sudden discovery of an excess stock of housing owned by the individuals in the closed economy leads to a *rise* in housing prices (relative to the initial steady state), the supply of labor is reduced on account of the wealth effect ("All those houses to enjoy!" as expressed by Phelps [2010]) but boosted by a positive wage effect (as wages are pulled up by higher housing prices). Equating the available stock of housing, $H(t)$, to aggregate housing demand, $H^d(t)$, in (17), and using (18) to substitute out for $v(t)$ in (17), we obtain $C^L(t) \equiv \bar{L} - L^s(t) = H(t)/[A\Lambda q(t)]$, so labor supply increases with $q(t)$ but decreases with $H(t)$. Are we able to determine unambiguously which effect dominates—the wealth effect or the wage effect? To answer this important question, it is helpful to characterize the general-equilibrium evolution of the closed economy in terms of another pair of variables—$H(t)$ (the stock of housing) and $C^L(t) \equiv \bar{L} - L^s(t)$ (the demand for leisure).

We rewrite (22) as

$$\dot{H}(t) = \Lambda[\bar{L} - C^L(t)] - \delta H(t). \tag{29}$$

Substituting $C^L(t) \equiv \bar{L} - L^s(t)$ in (17), using (18), setting $H^d(t) = H(t)$, and taking log, we obtain

$$\log C^L(t) = \log H(t) - \log A - \log \Lambda - \log q(t).$$

Taking the time derivative and then using (24) and (29), we obtain

$$\dot{C}^L(t) = \left\{ \frac{A\Lambda C^L(t)}{H(t)} - (\rho + \delta) \right\} C^L(t) - \left(\frac{A}{A+1} \right) \theta(\theta + \rho) \left(\frac{H(t)}{A\Lambda} \right). \tag{30}$$

The dynamic evolution of the economy is now summarized by (29) and (30) given an initial stock of housing $H(0)$.

Around the steady state, the slope of the stationary-H locus is given by

$$\left. \frac{dC^L(t)}{dH(t)} \right|_{\dot{H}(t)=0} = \frac{-\delta}{\Lambda} < 0. \tag{31}$$

In turn, the slope of the stationary-C^L locus is given by

$$\left. \frac{d^L C(t)}{dH(t)} \right|_{\dot{C}^L(t)=0} = \frac{\frac{A\Lambda C^L_{ss}}{(H_{ss})^2} + \left(\frac{1}{A+1} \right) \frac{\theta(\theta+\rho)}{\Lambda C^L_{ss}}}{\frac{A\Lambda}{H_{ss}} + \left(\frac{1}{A+1} \right) \frac{\theta(\theta+\rho) H_{ss}}{\Lambda (C^L_{ss})^2}} > 0. \tag{32}$$

The phase diagram characterizing the general-equilibrium evolution of the economy is shown in Figure 2, where it is readily checked that the system exhibits saddle-path stability. The saddle path is positively sloped in the H-C^L plane and lies in between the two stationary loci.

With the result that the saddle path is positively sloped in the H-C^L plane, it is now clear what the results are of a sudden discovery that the economy is bequeathed with an excess stock of housing (relative to the initial steady state). There is a sudden increase in the demand for leisure, C^L, upon the discovery of the excess housing stock. This is due both to a wealth effect of owning a higher stock of housing as well as a Hicks-Lucas-Rapping interest-rate effect as lower interest rates due to the excess housing stock lead agents to inter-temporally substitute towards consumption of leisure in the present. Since $C^L(t) \equiv \bar{L} - L^s(t)$, this implies that there is unambiguously a sudden decrease in labor supply. Combining this finding with our earlier result that the sudden discovery of excess housing stock leads to a sudden rise in housing prices, thus to a positive wage effect, this implies that the wealth effect and the Hicks-Lucas-Rapping interest-rate effect together overcome the positive wage effect. Another way of interpreting the result is to say that the lower interest rates brought about by the excess housing stock have two opposing effects: they help to lift up housing prices (relative to steady state) and thus pull up the demand-wage on one hand but discourage labor supply through inter-temporal substitution of leisure on the other hand. The positive wage effect, therefore, can only mitigate the slump in employment due to the wealth effect and the Hicks-Lucas-Rapping effect but does not dominate it. During the whole period during which the excess housing stock is worked off as the gross (flow) supply of housing falls below depreciation, employment is below normal (interpreted as the steady-state level absent another unforseen shock) though gradually recovering.

Now to complete the analysis, we study possible causes of the excess housing stock. In the modern economy fraught with uncertainty, it is not unnatural to suppose that the economy is initially disturbed by an anticipation of higher future housing prices not borne out in reality. Lemma 1 presents the list of parameters (the model's "fundamentals") that can raise the steady state level of housing prices. For concreteness, suppose that at time t_0 individuals in the economy form an expectation that at future time t_1 the subjective rate of time preference, ρ, will be reduced. As a result, as shown in Figure 3, the index of housing prices immediately jumps up from E_0 to E_1 after which it continues to steadily rise along E_1E_2. During this phase of a boom in the housing market, we infer from the corresponding path E_1E_2 in Figure 4 that there is a steady expansion of employment. Suppose that at time $t_2 \leq t_1$, it is realized that the earlier anticipation of a lower future ρ is not justified. Contrary to earlier expectations, fundamentals are in fact unchanged. In Figure 3, there is then an abrupt drop in $q(t)$ from E_2 to E_3 and correspondingly, in Figure 4, there is a sudden drop in employment as $C^L(t) \equiv \bar{L} - L^s(t)$ jumps up from E_2 to E_3 and the economy then finds itself with excess housing stock.

Proposition 1: Suppose that at time t_0 individuals in the economy form an expectation that at future time t_1, housing prices will be higher due to a shift in parameters that are the "fundamentals" determining long-run housing prices in the model. As a result, housing prices immediately jump up and continue a steady upward climb. During this phase of rising housing prices, aggregate employment is steadily increasing. When at time $t_2 \leq t_1$ it is realized that the earlier anticipation of a future parametric shift is not justified, there is an abrupt drop in the index of housing prices followed by a gradual decline to the initial steady state. With excess housing stock, the whole term structure of interest rates is shifted down in the post-bubble era, which serves to prop up housing prices and mitigate the employment slump.

3. The small open economy

We now open up the economy that we have been studying to unhindered international capital mobility and trade in housing services. We study two cases: (a) Foreign and domestic housing services are perfect substitutes; and (b) Foreign and domestic housing services are imperfect substitutes.

Foreign and domestic housing services are perfect substitutes

We suppose that the small open economy faces a world interest rate, r^*, that is parametrically given. We now have the condition that

$$r(t) = r^*.$$

With this condition, the general-equilibrium evolution of the economy can be summarized by the following system of four equations in the four variables: $H(t), q(t), W(t)$, and $C^L(t) \equiv \bar{L} - L^s(t)$ given initial $H(0)$.

$$\dot{H}(t) = \Lambda[\bar{L} - C^L(t)] - \delta H(t), \tag{33}$$

$$\dot{q}(t) = (r^* + \delta)q(t) - 1, \tag{34}$$

$$\dot{W}(t) = r^* W(t) + \Lambda q(t)\bar{L} - (A+1)\Lambda q(t) C^L(t), \tag{35}$$

$$\dot{C}^L(t) = \left[\frac{1}{q(t)} - (\rho + \delta)\right] C^L(t) - \left[\frac{\theta(\theta+\rho)}{A+1}\right]\left[\frac{W(t)}{\Lambda q(t)}\right]. \tag{36}$$

We study a linearized version of the dynamic system given by (33) to (36) around the steady-state $(H_{ss}, q_{ss}, W_{ss}, C^L_{ss})$ given by

$$[\dot{H}(t)\ \ \dot{q}(t)\ \ \dot{W}(t)\ \ \dot{C}^L(t)]' = \mathbf{A}[H(t) - H_{ss}\ \ q(t) - q_{ss}\ \ W(t) - W_{ss}\ \ C^L(t) - C^L_{ss}]',$$

where $[\cdots]'$ denotes a column vector, and the 4×4 matrix $\mathbf{A}$ contains the following elements:

$$\begin{aligned}
a_{11} &= -\delta < 0, \\
a_{12} &= 0, \\
a_{13} &= 0, \\
a_{14} &= -\Lambda < 0, \\
a_{21} &= 0, \\
a_{22} &= (r^* + \delta) > 0, \\
a_{23} &= 0, \\
a_{24} &= 0, \\
a_{31} &= 0, \\
a_{32} &= \Lambda\bar{L} - (A+1)\Lambda C^L_{ss}\ ?, \\
a_{33} &= r^* > 0, \\
a_{34} &= -(A+1)\Lambda q_{ss} < 0, \\
a_{41} &= 0, \\
a_{42} &= -\frac{C^L_{ss}}{q^2_{ss}} + \left[\frac{\theta(\theta+\rho)}{A+1}\right]\left[\frac{W_{ss}}{\Lambda q^2_{ss}}\right]\ ?, \\
a_{43} &= -\left[\frac{\theta(\theta+\rho)}{A+1}\right]\left[\frac{1}{\Lambda q_{ss}}\right] < 0, \\
a_{44} &= \left[\frac{1}{q_{ss}} - (\rho+\delta)\right] = r^* - \rho > 0 \text{ assumed.}
\end{aligned}$$

We let the eigenvalues of the system be given by λ_i with $i = 1, 2, 3, 4$. We can readily check that the determinant (equal to $\lambda_1\lambda_2\lambda_3\lambda_4 \equiv a_{11}a_{22}a_{33}a_{44}$) is negative so there are either one negative root plus three positive roots or three negative roots plus one positive root. If there is only one negative root, we let it be denoted by λ_1. If there are three negative roots, we denote the three negative roots by λ_1, λ_2 and λ_3 and assume that λ_1 is the dominant eigenvalue, that is, $0 > \lambda_1 > \lambda_2 > \lambda_3$. Following the method of dominant eigenvalue proposed by Calvo (1987), we infer that the system will converge to a ray which is associated with λ_1.

Let the eigenvector associated with λ_1 be given by $[1 \;\; x_{21} \;\; x_{31} \;\; x_{41}]'$. We obtain the system of equations:

$$\lambda_1 - a_{11} - a_{14}x_{41} = 0, \tag{37}$$

$$(\lambda_1 - a_{22})x_{21} = 0, \tag{38}$$

$$-a_{32}x_{21} + (\lambda_1 - a_{33})x_{31} - a_{34}x_{41} = 0, \tag{39}$$

$$-a_{42}x_{21} - a_{43}x_{31} + (\lambda_1 - a_{44})x_{41} = 0. \tag{40}$$

From (38), we see that with $(\lambda_1 - a_{22}) < 0$, we have $x_{21} = 0$. Moreover, from (37), we have

$$x_{41} = \frac{\lambda_1 - a_{11}}{a_{14}} > 0 \text{ for small } \delta.$$

We have the following lemma:

Lemma 2: $x_{21} = 0$, and, for small δ, $x_{41} > 0$.

Using Lemma 2, we can draw a diagram like Figure 5. In the top panel, we have a horizontal line whose intercept is given by $q_{ss} = (r^* + \delta)^{-1}$ while the bottom panel shows a positively-sloped asymptotic adjustment path between the index of housing prices and the stock of housing. In this setting, if the economy is initially at E_0 and is suddenly bequeathed with an excess stock of housing, $q(t)$ does not change because the excess stock of housing can be rented to foreigners without affecting housing rentals. The lift to wages and thus employment found in the closed economy due to a downward shift of the term structure of interest rates is now no longer operative. The excess stock of housing would have no effect on employment on account of the wage effect alone. However, as there is a wealth effect, we observe from the bottom panel in Figure 5 that aggregate employment immediately falls, as $C^L(t) \equiv \bar{L} - L(t)$ jumps up, and gradually recovers.

In the small open economy here, an expectation formed at current time t_0 that the world interest rate at future time t_1 will be reduced can precipitate a sudden rise in the index of housing prices from E_0 to E_1 followed by a gradual rise along E_1E_2. If at time $t_2 \leq t_1$, it is realized that the earlier expectation of higher future housing prices is not based upon "fundamentals" (here the parameters r^* and δ), there is a sudden drop of housing prices from E_2 to E_3 coinciding with the initial steady-state value of q_{ss}. Employment falls and follows the path E_3E_0.

Proposition 2: Suppose that at time t_0 individuals in the small open economy form an expectation that at future time t_1, housing prices will be higher due to a shift in parameters that are the "fundamentals" determining long-run housing prices in the model. As a result, housing prices immediately jump up and continue a steady upward climb. During this phase of rising housing prices, aggregate employment is steadily increasing. When at time $t_2 \leq t_1$ it is realized that the earlier anticipation of a future parametric shift is not justified, there is an abrupt drop in the index of housing prices to the initial steady state. With no change in the interest rate, there is no longer a mechanism to prop up housing prices to mitigate the employment slump found to be operative in the closed economy.

Foreign and domestic housing services are imperfect substitutes

Now, we assume that foreign and domestic housing services though tradable are not perfectly substitutable. We set foreign rentals, R^*, to unity and assume that the world interest rate, r^*, is constant in terms of foreign units of housing services.

At time 0, the individual's intertemporal optimization problem can be written as

$$\begin{aligned}
&\text{Maximize} \int_0^\infty \{A^{-1}[\log(\bar{L} - l(s,t))] + \log[(h^h(s,t))^\gamma (h^f(s,t))^{1-\gamma}] + B\} \exp^{-(\theta+\rho)t}\, dt \\
&\text{subject to } \dot{w}(s,t) = [r^* + \theta]w(s,t) + v(t)l(s,t) - R(t)h^h(s,t) - h^f(s,t), \\
&\text{given } w(s,0).
\end{aligned}$$

Here $h^d(s,t)$ is consumption of domestic housing services with housing rental of $R(t)$ and $h^f(s,t)$ is consumption of foreign housing services with housing rental of one.

Let $e(s,t) \equiv R(t)h^h(s,t) + h^f(s,t)$. Then, static optimization gives

$$R(t)h^h(s,t) = \gamma e(s,t), \tag{41}$$

$$h^f(s,t) = (1-\gamma)e(s,t). \tag{42}$$

Using (41) and (42) in the intertemporal optimization problem, we can rewrite the problem as

$$\begin{aligned}
&\text{Maximize } \int_0^\infty \{A^{-1}[\log(\bar{L} - l(s,t))] + \log\left[\frac{\gamma^\gamma(1-\gamma)^{1-\gamma}}{(R(t))^\gamma}\right] + \log e(s,t) + B\} \exp^{-(\theta+\rho)t} dt \\
&\text{subject to } \dot{w}(s,t) = [r^* + \theta]w(s,t) + v(t)\bar{L} - v(t)(\bar{L} - l(s,t)) - e(s,t), \\
&\text{given } w(s,0).
\end{aligned}$$

Solving the individual's intertemporal problem gives the following optimal conditions:

$$\frac{A^{-1}}{\bar{L} - l(s,t)} = v(t)\mu(s,t), \tag{43}$$

$$\frac{1}{e(s,t)} = \mu(s,t), \tag{44}$$

$$\frac{\dot{\mu}(s,t)}{\mu(s,t)} = -[r(t) - \rho], \tag{45}$$

$$\lim_{T\to\infty} \mu(s,T)w(s,T)\exp^{-(\theta+\rho)T} = 0, \tag{46}$$

where $\mu(s,t)$ is the co-state variable. Combining (43) and (44), and eliminating $\mu(s,t)$, we obtain

$$e(s,t) = Av(t)[\bar{L} - l^s(s,t)]. \tag{47}$$

Using (47), the individual dynamic budget constraint can be written as

$$\dot{w}(s,t) = (r^* + \theta)w(s,t) + v(t)\bar{L} - \left(\frac{A+1}{A}\right)e(s,t),$$

and the individual expenditure function can be written as

$$e(s,t) = (\theta+\rho)\left(\frac{A}{A+1}\right)\left[w(s,t) + \int_t^\infty v(\kappa)\bar{L}\exp^{-\int_t^\kappa [r(\nu)+\theta]d\nu} d\kappa\right].$$

Aggregating across cohorts in (47), and using $v(t) = \Lambda q(t)$, we obtain, after re-arranging,

$$L^s(t) = \bar{L} - \frac{E(t)}{A\Lambda q(t)}, \tag{48}$$

where $E(t) = \int_{-\infty}^{t} \theta \exp^{-\theta(t-s)} e(s,t)ds$. We let foreigners' demand for our home housing services be given by $(1-\gamma)E^*(t)/R(t)$, where $E^*(t)$ is aggregate foreign expenditure. The domestic demand for our home housing services is given by $\gamma E(t)/R(t)$. Thus

$$\text{Aggregate foreigners' and domestic demand for home's housing services} = \frac{\gamma E(t) + (1-\gamma)E^*(t)}{R(t)}.$$

Equating total demand for home's housing services to total stock of housing gives

$$R(t) = \frac{\gamma E(t) + (1-\gamma)E^*(t)}{H(t)}. \tag{49}$$

In what follows, we treat $E^*(t)$ as exogenous and assume a constant flow.

The general-equilibrium evolution of the economy can be summarized by the following system of four equations in the four variables: $H(t), q(t), W(t)$, and $E(t)$ given initial $H(0)$.

$$\dot{H}(t) = \Lambda\bar{L} - \frac{E(t)}{Aq(t)} - \delta H(t), \tag{50}$$

$$\dot{q}(t) = (r^* + \delta)q(t) - \frac{\gamma E(t) + (1-\gamma)E^*}{H(t)}, \tag{51}$$

$$\dot{W}(t) = r^* W(t) + \Lambda q(t)\bar{L} - \left(\frac{A+1}{A}\right) E(t), \tag{52}$$

$$\dot{E}(t) = (r^* - \rho)E(t) - \theta(\theta+\rho)\left(\frac{A+1}{A}\right) W(t). \tag{53}$$

We study a linearized version of the dynamic system given by (50) to (53) around the steady-state $(H_{ss}, q_{ss}, W_{ss}, E_{ss})$ given by

$$[\dot{H}(t) \;\; \dot{q}(t) \;\; \dot{W}(t) \;\; \dot{E}(t)]' = \mathbf{A}[H(t) - H_{ss} \;\; q(t) - q_{ss} \;\; W(t) - W_{ss} \;\; E(t) - E_{ss}]',$$

where $[\cdots]'$ denotes a column vector, and the 4×4 matrix $\mathbf{A}$ contains the following elements:

$$
\begin{aligned}
a_{11} &= -\delta < 0, \\
a_{12} &= \frac{E_{ss}}{Aq_{ss}^2} > 0, \\
a_{13} &= 0, \\
a_{14} &= -\frac{1}{Aq_{ss}} < 0, \\
a_{21} &= \frac{\gamma E_{ss} + (1-\gamma)E^*}{H_{ss}^2} > 0, \\
a_{22} &= (r^* + \delta) > 0, \\
a_{23} &= 0, \\
a_{24} &= -\frac{\gamma}{H_{ss}} < 0, \\
a_{31} &= 0, \\
a_{32} &= \Lambda \bar{L} > 0, \\
a_{33} &= r^* > 0, \\
a_{34} &= -\left(\frac{A+1}{A}\right) < 0, \\
a_{41} &= 0, \\
a_{42} &= 0, \\
a_{43} &= -\left(\frac{A+1}{A}\right)\theta(\theta+\rho) < 0, \\
a_{44} &= r^* - \rho > 0 \text{ assumed.}
\end{aligned}
$$

We let the eigenvalues of the system be given by λ_i with $i = 1, 2, 3, 4$. The determinant is equal to $\lambda_1\lambda_2\lambda_3\lambda_4 \equiv a_{11}a_{22}a_{33}a_{44} + a_{21}a_{32}a_{43}a_{14} - a_{12}a_{21}a_{43}a_{34}$ and can be positive or negative. We will treat the case where the determinant is negative so there are either one negative root plus three positive roots or three negative roots plus one positive root. If there is only one negative root, we let it be denoted by λ_1. If there are three negative roots, we denote the three negative roots by λ_1, λ_2 and λ_3 and assume that λ_1 is the dominant eigenvalue, that is, $0 > \lambda_1 > \lambda_2 > \lambda_3$. Following the method of dominant eigenvalue proposed by Calvo (1987), we infer that the system will converge to a ray which is associated with λ_1.

Let the eigenvector associated with λ_1 be given by $[1 \;\; x_{21} \;\; x_{31} \;\; x_{41}]'$. We obtain the system of equations:

$$\begin{aligned} \lambda_1 - a_{11} - a_{12}x_{21} - a_{14}x_{41} &= 0, & (54) \\ -a_{21} + (\lambda_1 - a_{22})x_{21} - a_{24}x_{41} &= 0, & (55) \\ -a_{32}x_{21} + (\lambda_1 - a_{33})x_{31} - a_{34}x_{41} &= 0, & (56) \\ -a_{43}x_{31} + (\lambda_1 - a_{44})x_{41} &= 0. & (57) \end{aligned}$$

From (54) and (55), we obtain

$$x_{21} = \frac{a_{21} + \left(\frac{a_{24}}{a_{14}}\right)(\lambda_1 - a_{11})}{(\lambda_1 - a_{22}) + \frac{a_{24}a_{12}}{a_{14}}},$$

which can either be positive or negative. From (55), we have

$$x_{41} = \frac{-a_{21} + (\lambda_1 - a_{22})x_{21}}{a_{24}},$$

which can also be positive or negative. We have the following lemma:

Lemma 3: x_{21} and x_{41} can be either positive or negative.

Using Lemma 3, we can depict a worst-case scenario in Figure 6. In the top panel, we have a negatively-sloped asymptotic adjustment path between the index of housing prices and the stock of housing. Then in the bottom panel, we have a positively-sloped asymptotic adjustment path between the domestic expenditure and the stock of housing. In this setting, if the economy is initially at E_0 and is suddenly bequeathed with an excess stock of housing, $q(t)$ drops as housing rentals, $R(t)$, in home decline due to excess supply in the housing market. Relative to foreign housing rentals (normalized to one), the excess housing stock leads to a real exchange rate depreciation. The real exchange rate depreciation brought about by the excess housing stock has a depressing effect on housing prices as

$$q(t) = \int_t^\infty R(\kappa) \exp^{-(r^*+\delta)(\kappa-t)} \, d\kappa.$$

From (48), we note that aggregate employment falls both because $q(t)$ is down and also because $E(t)$ is up. The real exchange rate depreciation now exerts a negative wage effect as $q(t)$ falls below steady state. Thus employment falls both because of a wealth effect as well as a negative wage effect.

As in the case of perfectly substitutable foreign and domestic housing services, an expectation formed at current time t_0 that the world interest rate at future time t_1 will be reduced can precipitate a sudden rise in the index of housing prices from E_0 to E_1 followed by a gradual rise along E_1E_2. If at time $t_2 \leq t_1$, it is realized that the earlier expectation of higher future housing prices is not based upon "fundamentals" (such as the parameter r^*), there is a sudden drop of housing prices from E_2 to E_3, a level below the initial and final steady-state value of q_{ss}. Employment falls and follows the path E_3E_0.

Proposition 3: Suppose that at time t_0 individuals in the small open economy form an expectation that at future time t_1, housing prices will be higher due to a shift in parameters that are the "fundamentals" determining long-run housing prices in the model. As a result, housing prices immediately jump up and continue a steady upward climb. During this phase of rising housing prices, aggregate employment is steadily increasing. When at time $t_2 \leq t_1$ it is realized that the earlier anticipation of a future parametric shift is not justified, there is an abrupt drop in the index of housing prices possibly to a level below the steady state as the economy goes through a period of real exchange rate depreciation as the excess housing stock is run down. The real exchange rate depreciation worsens employment as it has a depressing effect on housing prices and thus a depressing effect on wages.

4. Conclusion

We have shown how over-investment, in causing the housing stock to get ahead of itself, has the result that new housing construction will for a period be *below* its normal path. Correspondingly, employment will stay for a period below its normal path, a transient slump after the false boom. This lends theoretical support for the Austrian thesis that when gathering euphoria brings over-investment, society must pay the price of a temporary slump. In the closed economy case, the downward shift of the term structure of interest rates due to the excess housing stock props up housing prices above the normal steady-state level, so the drop of housing prices "undershoots." Although this transient elevation of housing prices has a positive demand-wage effect on employment, the wealth effect from owning a higher housing stock and a negative Hicks-Lucas-Rapping effect of lower interest rates dominate, so employment drops initially to a below-normal level. The slump gradually subsides as the housing overhang wears off.

There are two possibilities in the small open economy. If housing services are instantaneously tradeable and perfect substitutes for foreign ones, so purchasing power parity holds, the end of the bubble causes housing prices to drop precisely to the steady-state level. Since there is no undershooting, the wealth effect of the housing overhang is unopposed and the slump is deeper. If domestic and foreign housing services are imperfect substitutes, the country will suffer a period with a weak real exchange rate, thus to a drop of housing prices that "overshoots" the normal level. Here the slump in employment is worsened by the exaggerated fall in housing prices below the steady-state level.

References

Benhabib, Jess, Richard Rogerson, and Randall Wright, 1991, "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, 99, December, 1166-87.

Blanchard, Olivier, 1985, "Debts, Deficits and Finite Horizons," *Journal of Political Economy*, 93, April, 223-47.

Calvo, Guillermo, 1987. "Real Exchange Rate Dynamics with Nominal Parities: Structural Change and Overshooting," *Journal of International Economics*, 22, 141-55.

Hoon, Hian Teck and Edmund S. Phelps, 1996, "Payroll Taxes and VAT in a Labor-Turnover Model of the 'Natural Rate'," *International Tax and Public Finance*, 3, 369-83.

Hoon, Hian Teck, forthcoming, "Payroll Taxes, Wealth and Employment in Neoclassical Theory: Neutrality or Non-Neutrality?" in Edmund S. Phelps and Hans Werner Sinn (eds.), *The Performance of Continental Economies*, Cambridge, MA: MIT Press.

Obstfeld, Maurice, 1989, "Fiscal Deficits and Relative Prices in a Growing World Economy," *Journal of Monetary Economics*, 23(3), May, 461-84.

Phelps, Edmund, 2010, "The Slump, the Recovery, and the 'New Normal'," Center Working Paper No. 61, June, Center on Capitalism and Society, Columbia University.
Weil, Philippe, 1989, "Overlapping Families of Infinitely-Lived Agents," *Journal of Public Economics*, 38, 183-98.
Zoega, Gylfi, 2010, "The Financial Crisis: Joblessness and Investmentlessness," Center Working Paper No. 62, July, Center on Capitalism and Society, Columbia University.

Appendix

We first focus on an individual's choice of his time spent in market work, non-market housework, and time for leisure. We explicitly model the choice of time spent in three activities: the market sector, non-market housework, and leisure. Building upon Benhabib, Rogerson and Wright (1991), we suppose that the period individual utility function is given by

$$\begin{aligned} U &= \log \hat{c} + A' \log[\bar{L} - l_m - l_n] + B', \ \text{ if } l_m > 0 \\ &= \log \hat{c} + A' \log[\bar{L} - l_m - l_n], \qquad \text{ if } l_m = 0, \end{aligned}$$

where $A', \ B' > 0$ and $\hat{c} \equiv h^{\mu} c_n^{1-\mu}$, $0 < \mu < 1$. Here, $\bar{L}$ is time endowment, l_m is time spent working in the market sector, l_n is time spent in non-market housework, h is consumption of housing services, and c_n is consumption of the home produced non-market good. We assume that the non-market good is produced according to $c_n = s_n l_n; \ s_n > 0$. Notice that as in Benhabib, Rogerson and Wright (1991), we suppose that working in the market sector gives positive direct utility, presumably because one enjoys certain social interactions and types of mental stimulation at the work place that one does not get by devoting all of one's time to leisure and homework. We assume that there is a fixed positive utility value from working in the market sector (given by B') that is independent of the actual number of hours worked. In contrast, the utility value derived from housework comes indirectly from consuming the home-produced good generated by the time input into the non-market sector.

To ensure that every living person in the economy spends a positive amount of time working in the market in order to facilitate aggregation, we make the assumption that the direct utility value from spending a positive amount of time in the market is sufficiently large.

(See Assumption 1 below.)

The agent maximizes

$$\int_t^\infty \{\log[(h(s,\kappa))^\mu (c_n(s,\kappa))^{1-\mu}] + A' \log[\bar{L} - l_n(s,\kappa) - l_m(s,\kappa)] + B'\} \exp^{-(\theta+\rho)(\kappa-t)} \, d\kappa$$

subject to

$$\begin{aligned} c_n(s,t) &= s_n l_n(s,t), \\ \frac{dw(s,t)}{dt} &= [r(t)+\theta]w(s,t) + v(t)l_m(s,t) - R(t)h(s,t), \end{aligned}$$

and a transversality condition that prevents agents from going indefinitely into debt. As in Blanchard (1985), agents save or dissave by buying or selling actuarial bonds, that is, bonds that are cancelled by death. Here, ρ is the subjective rate of time discount, θ is the constant instantaneous probability of death so θ^{-1} is the expected remaining life, $w(s,t)$ is non-human wealth at time t of an agent born at time s, $R(t)$ is housing rental, and $v(t)$ is wage rate.[5] The rate of interest on actuarial bonds is $r(t)+\theta$.

From the optimal choice of h, l_m, and l_n, we obtain, after some manipulation, the following two relationships:

$$\frac{\mu v}{h} = \frac{A'}{\bar{L} - l_n - l_m}, \tag{58}$$

$$\frac{(1-\mu)s_n}{c_n} = \frac{A'}{\bar{L} - l_n - l_m}. \tag{59}$$

Using these two equations to get $c_n/h = (1-\mu)s_n(\bar{L} - l_m)[(A' + (1-\mu))h]^{-1}$, and using $c_n = s_n l_n$, we further obtain $l_n = (1-\mu)(A')^{-1}[\bar{L} - l_n - l_m]$. We can then eliminate l_n and c_n and write the individual's intertemporal optimization problem simply as

$$\text{Maximize} \quad \int_t^\infty \{\log h(s,\kappa) + A^{-1}\log[\bar{L} - l_m(s,\kappa)] + B\} \exp^{-(\theta+\rho)(\kappa-t)} \, d\kappa$$

subject to

$$\frac{dw(s,t)}{dt} = [r(t)+\theta]w(s,t) + v(t)l_m(s,t) - R(t)h(s,t), \tag{60}$$

[5] We assume that the wage per hour worked in the market is independent of the age of the agent.

where

$$
\begin{aligned}
A^{-1} &\equiv \mu^{-1}[A' + (1-\mu)], \\
B &\equiv \mu^{-1}(1-\mu)\log\left[\frac{(1-\mu)s_n}{A' + (1-\mu)}\right] + \mu^{-1}A'\log\left[\frac{A'}{A' + (1-\mu)}\right] + B'\mu^{-1}.
\end{aligned}
$$

We make a notational change in the main text and drop the "m" subscript to denote market work. Thus, $l(s,t)$ replaces $l_m(s,t)$ with the understanding that $l(s,t)$ is the individual's supply of labor to the market sector.

We make the following assumption:

Assumption 1: $B > A^{-1}[\log \bar{L} - \log(\bar{L} - 0^+)]$.

Under Assumption 1, a very wealthy individual who might have chosen to retire in a model without a positive utility value from market work spends a very small positive amount of time working in the market ($l_m = 0^+ > 0$) given the positive utility value of market work compared to housework in our model.

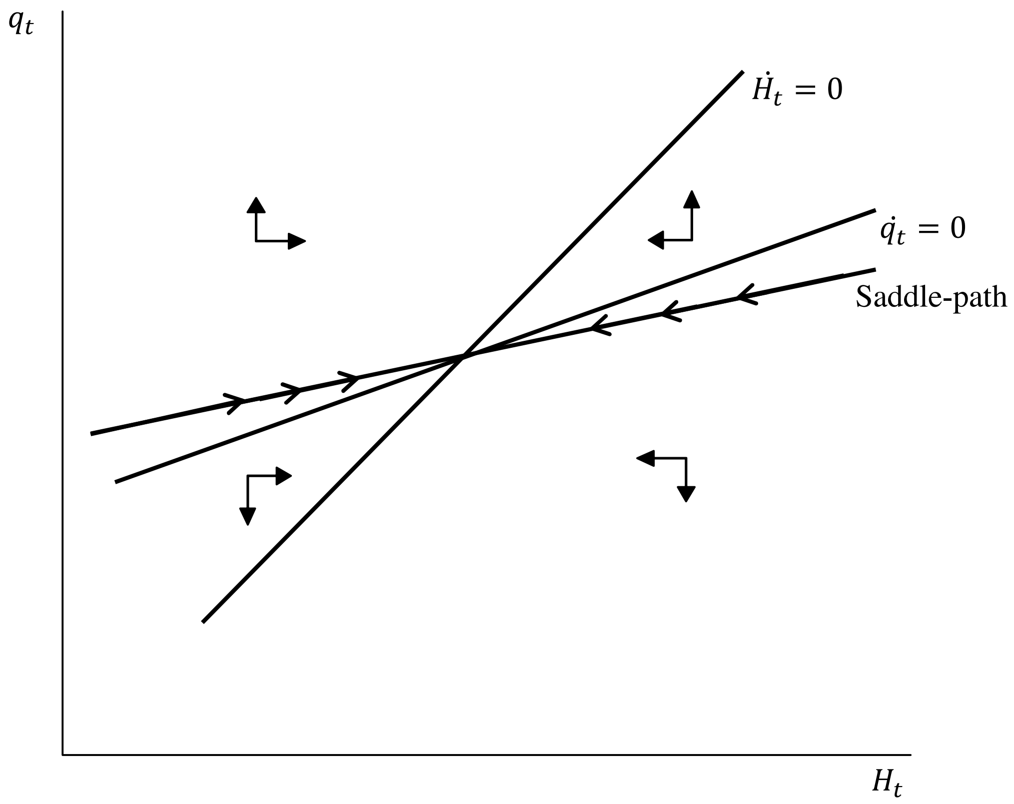

Figure 1: Saddle-path stability in *H*-*q* plane in closed economy

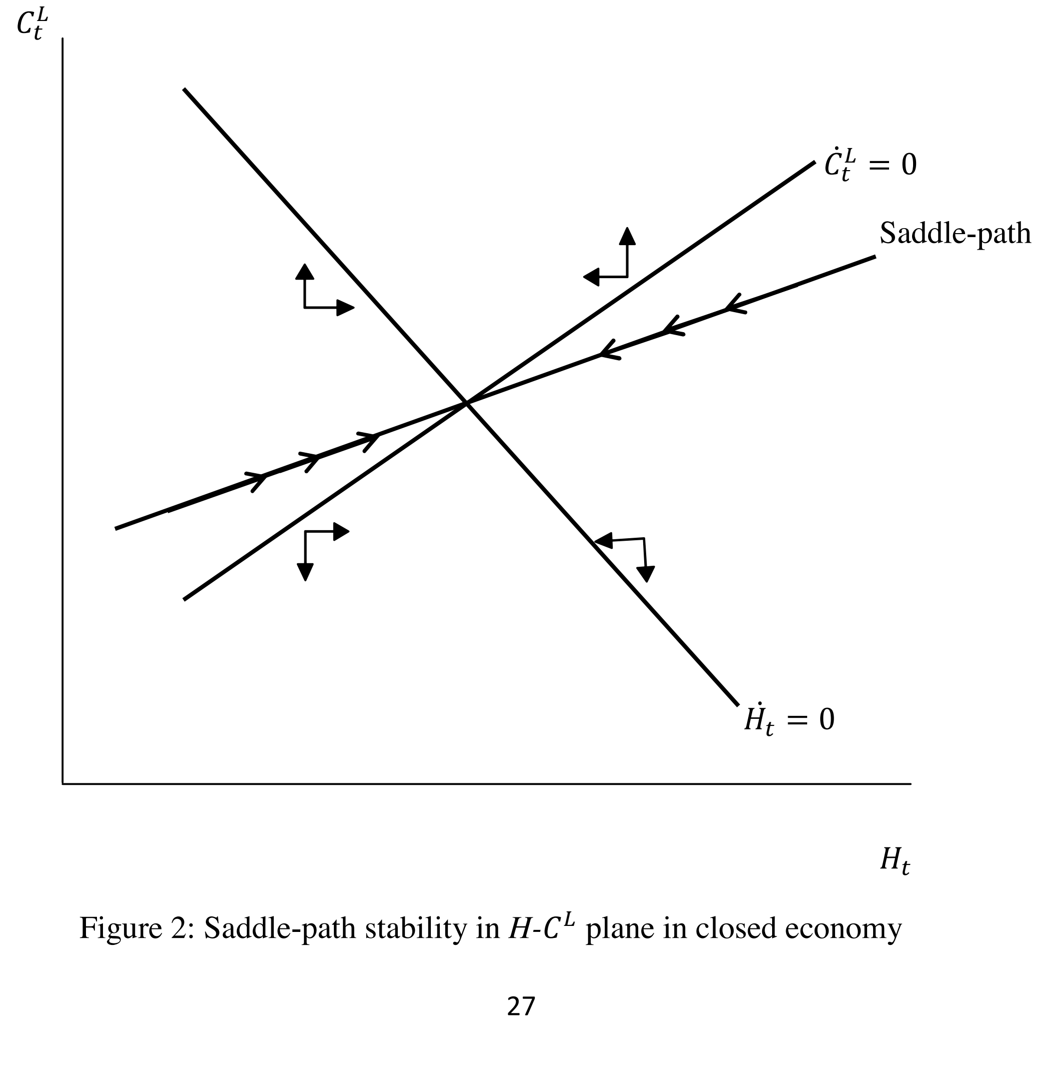

Figure 2: Saddle-path stability in H-C^L plane in closed economy

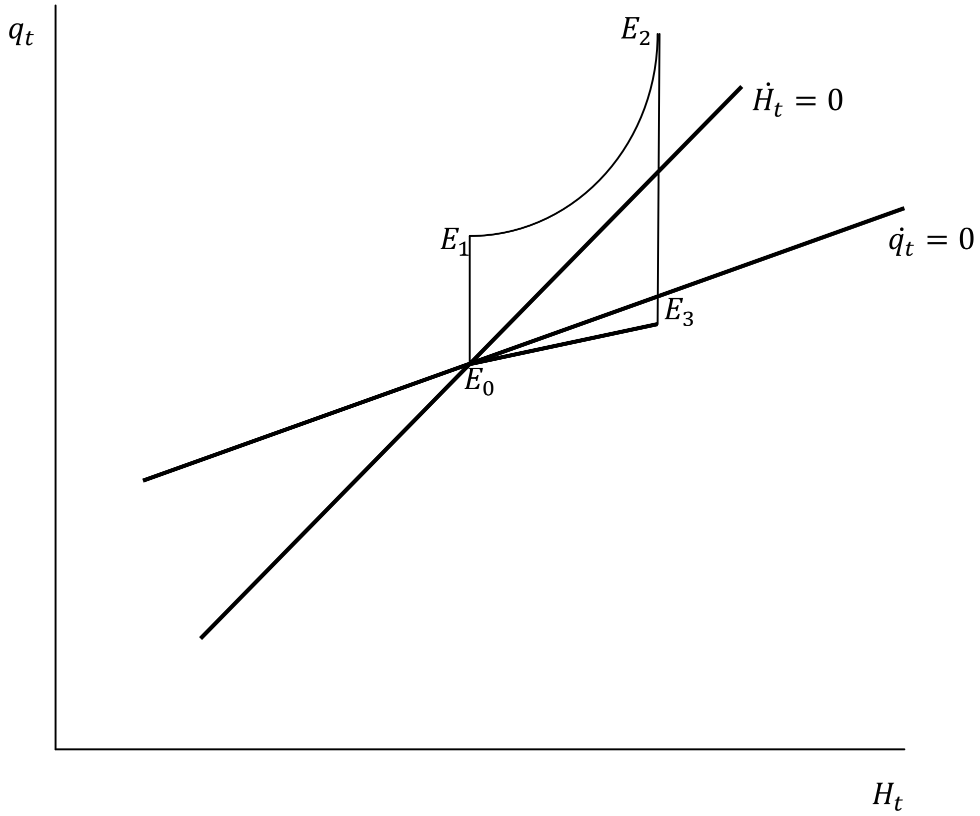

Figure 3: Housing bubble and aftermath in closed economy in *H*-*q* plane

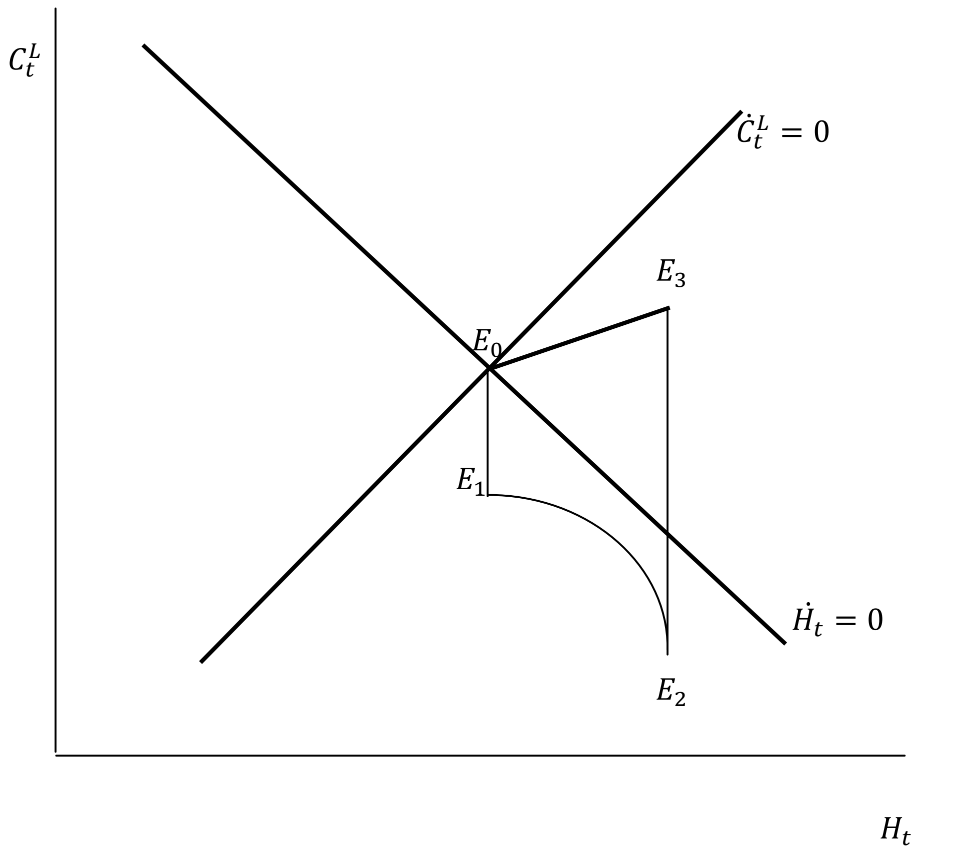

Figure 4: Housing bubble and aftermath in closed economy in H-C^L plane

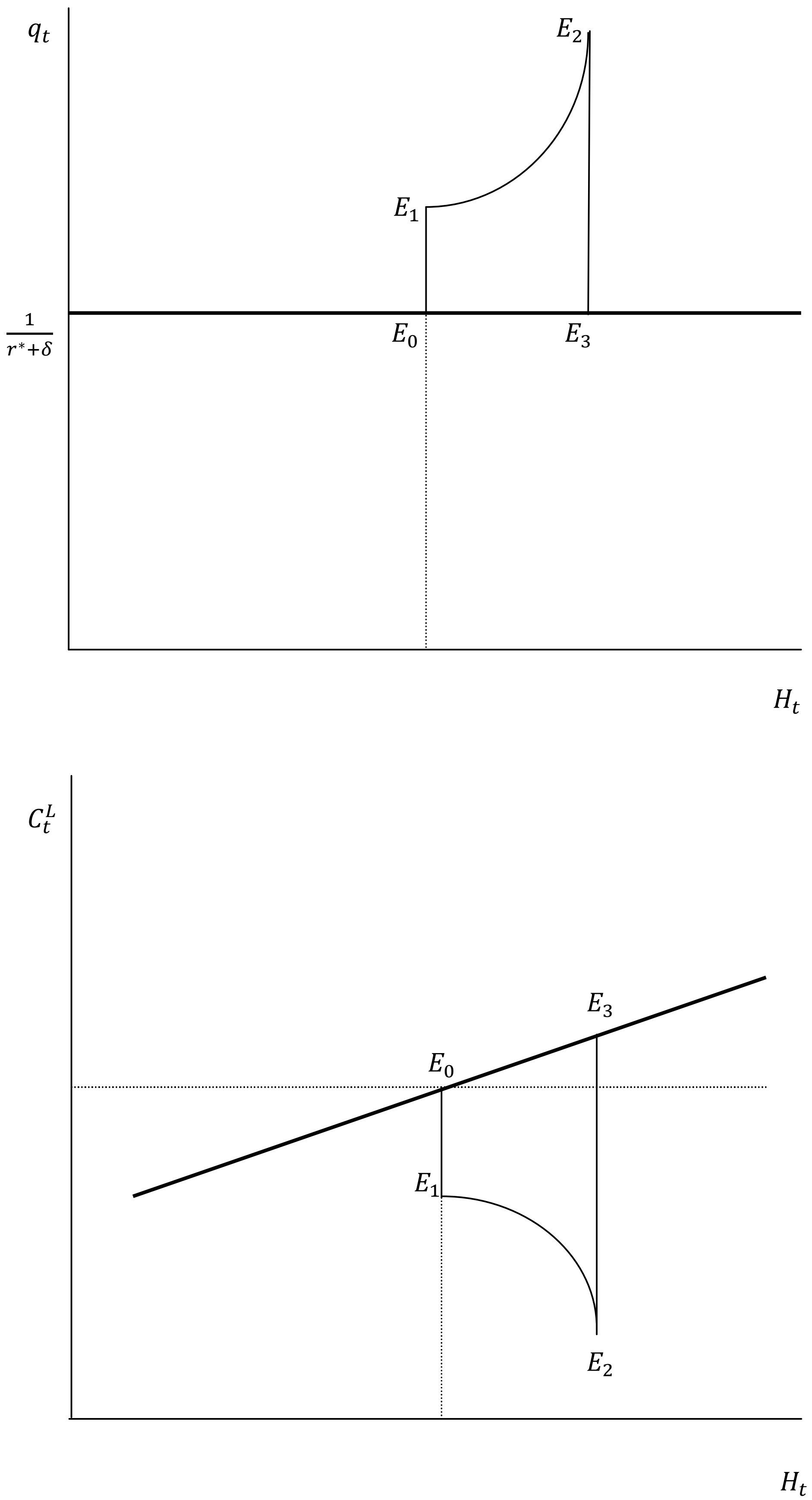

Figure 5: Housing bubble and aftermath in small open economy with perfectly substitutable foreign and domestic housing services

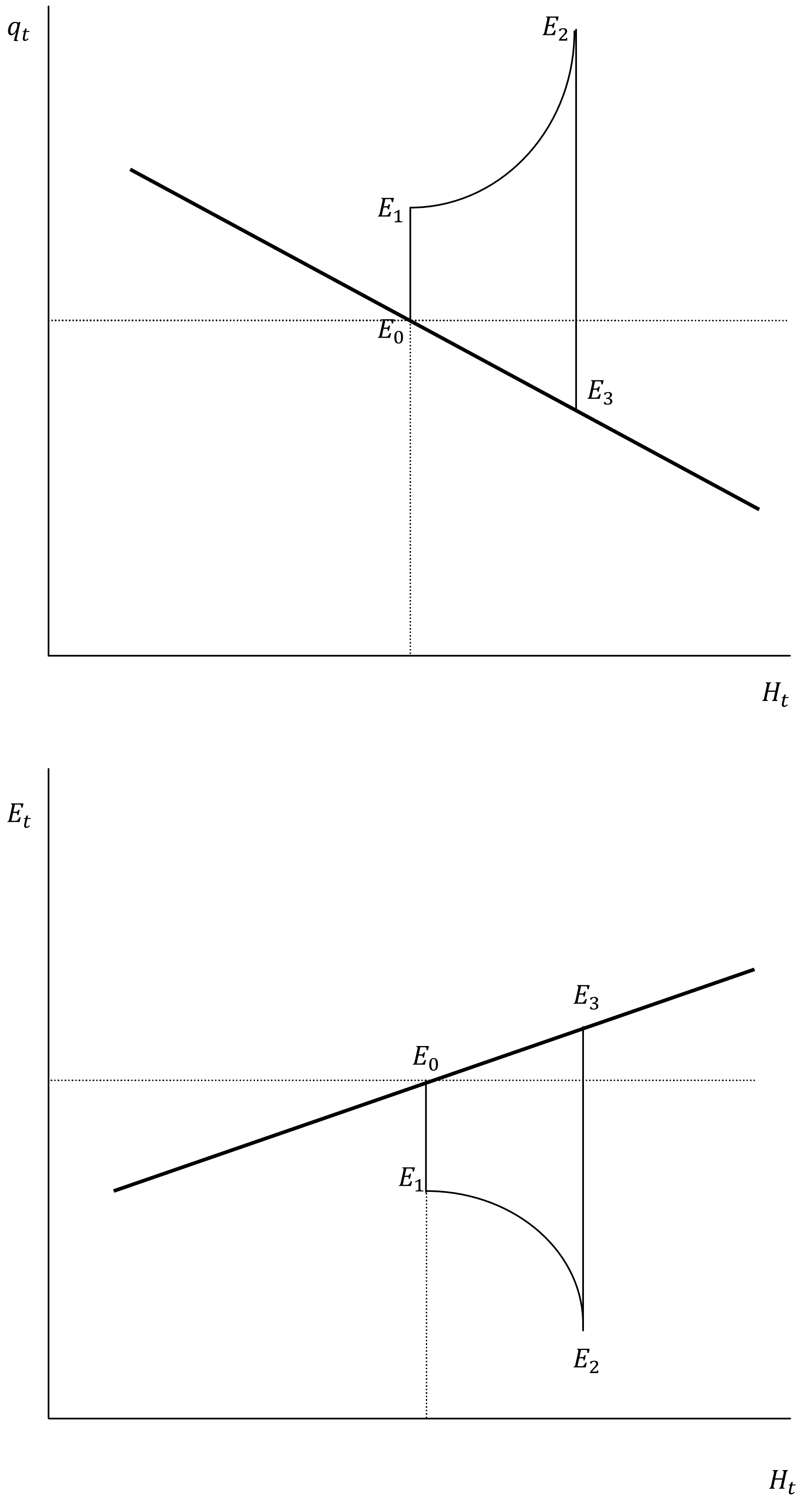

Figure 6: Housing bubble and aftermath in small open economy with imperfectly substitutable foreign and domestic housing services